

Alteration in Non-Classicality of Light on Passing Through a Linear Polarization Beam Splitter

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We observe the polarization squeezing in the mixture of a two mode squeezed vacuum and a simple coherent light through a linear polarization beam splitter. Squeezed vacuum not being squeezed in polarization, generates polarization squeezed light when superposed with coherent light. All the three Stokes parameters of the light produced on the output port of polarization beam splitter are found to be squeezed and squeezing factor also depends upon the parameters of coherent light.

INTRODUCTION

In classical optics, the state of polarization of a light beam is visualized by Stokes vector on the Poincare sphere [1, 2] and it is determined by four Stokes parameters S_0 and $\mathbf{S} = (S_1, S_2, S_3)$, following the relation $\mathbf{S}^2 = S_1^2 + S_2^2 + S_3^2$. These Stokes parameters involve coherence functions [3] of order (1,1) and it has been realized that these are insufficient to describe polarization completely as $\mathbf{S} = 0$ does not represent only unpolarized light [4]. But, these parameters still remain important because of their role in non-classicalities associated with polarization. Quantum mechanical analogue of Stokes parameters can also be defined to characterize quantum nature of polarization. These quantum Stokes operators hermitian in nature and act as observables for the system. These hermitian Stokes operators can be defined as the quantum versions of their classical counterparts and these are given by

$$\hat{S}_{0,1} = \hat{a}_x^\dagger \hat{a}_x \pm \hat{a}_y^\dagger \hat{a}_y, \quad \hat{S}_2 + i\hat{S}_3 = 2\hat{a}_x^\dagger \hat{a}_y, \quad (1)$$

where $\hat{a}_{x,y}, \hat{a}_{x,y}^\dagger$ refer to the photon annihilation and creation operators respectively of the two orthogonal polarization modes x and y satisfying the commutation relations $[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{jk}$ for $j, k = x, y$. The mean value of the radius of quantum Poincare sphere is given by square root of expectation value of either side of the equation

$$\hat{S}_1^2 + \hat{S}_2^2 + \hat{S}_3^2 = \hat{S}_0^2 + 2\hat{S}_0. \quad (2)$$

Note that, Stokes operators do not commute with each other and hence this equation has an extra term $2\hat{S}_0$ on the right hand side. Thus, a quantum polarization state is described by the four Stokes operators, \hat{S}_0 which is the total photon number operator denoting the beam intensity and $\hat{\mathbf{S}} = \hat{S}_1, \hat{S}_2, \hat{S}_3$ with commutation relations

$$[\hat{S}_0, \hat{S}_j] = 0, \quad [\hat{S}_j, \hat{S}_k] = 2i \sum_l \epsilon_{jkl} \hat{S}_l, \quad (3)$$

following the SU(2) algebra. Here, ϵ_{jkl} is Levi-Civita symbol for $(j, k, l = 1, 2, \text{ or } 3, j \neq k \neq l \neq j)$. These relations are parallel to the commutation relations for components of the angular momentum operator. These non-zero commutators

show that simultaneous exact measurement of the quantities represented by these Stokes operators are impossible and the following uncertainty relations hold

$$V_j V_k \geq \langle \hat{S}_l \rangle^2, \quad V_j \equiv \langle \hat{S}_j^2 \rangle - \langle \hat{S}_j \rangle^2. \quad (4)$$

Here V_j stands for the variance $\langle \hat{S}_j^2 \rangle - \langle \hat{S}_j \rangle^2$ of the quantum Stokes operators \hat{S}_j .

Polarization squeezing is a non-classicality similar to ordinary squeezing, and is defined almost in a similar fashion using Stokes operators as the continuous variables for the system describing polarization. Polarization squeezing first introduced by Chirkin et al. [5] was initially defined using the commutation relations followed by the Stokes operators. Later, this definition was modified by Heersink et al. [6] taking into account the uncertainty relations followed by these Stokes operators and generalized by Luis and Korolkova [7] for a general component of Stokes operator vector. The authors have written the criterion for polarization squeezing for a general component of Stokes vector operator \hat{S}_n along the unit vector \mathbf{n} [8–10] in the following form

$$V_n \equiv \langle \Delta \hat{S}_n^2 \rangle < |\langle \hat{S}_{n_\perp} \rangle|_{max} = \sqrt{|\langle \hat{\mathbf{S}} \rangle|^2 - \langle S_n \rangle^2} \quad (5)$$

arguing that, there are infinite directions \mathbf{n}_\perp for a given component \hat{S}_n and therefore one needs to consider the maximum possible value of $|\langle \hat{S}_{n_\perp} \rangle|$. All of the above definitions have been used in different studies on polarization squeezing [11–13] in the order of improvement considering the uncertainty relations and characterization of polarization squeezing in a general component of Stokes operator vector. In our study, we use the criterion in Eq. (5) for polarization squeezing, which is the most general and based on the actual uncertainty relations. Squeezing factor \mathcal{S}_n and degree of squeezing \mathcal{D}_n to measure polarization squeezing can be defined as

$$\mathcal{S}_n = \frac{V_n}{\sqrt{|\langle \hat{\mathbf{S}} \rangle|^2 - \langle \hat{S}_n \rangle^2}}, \quad \mathcal{D}_n = 1 - \mathcal{S}_n, \quad (6)$$

respectively. Non-classicalities are seen when $1 > \mathcal{S}_n > 0$ and the degree of squeezing \mathcal{D}_n lies between 0 and 1.

Since, quantum Stokes operators and non-classical polarization can be used for quantum information protocols in quantum communication, it is important to study polarization squeezing in such systems. Another convenience reason being the easy measurement of Stokes parameters using linear optical elements, polarization squeezing is easy to experimentally measure. The direct measurement schemes are developed methods for measuring these parameters and they preserve quantum noise property.

In the present paper, we study a process where linear beam splitter mixes coherent light with a two mode squeezed vacuum and it is observed that, the output beam from the beam splitter exhibits polarization squeezing. Illustration in Fig. 1 shows the superposition on linear polarization beam splitter. The input non-classical light is a two mode squeezed vacuum which does not show polarization squeezing. The linear beam splitter can not convert classical light beams into non-classical though nonlinear beam splitters do. The polarization squeezing at the output port **3** therefore shows that, the input non-classical beam gives non-classicality in the output.

If the two mode coherent light beam incident at ports **1** and two mode squeezed vacuum at port **2**, are represented by annihilation operators \hat{a}_x, \hat{a}_y and \hat{b}_x, \hat{b}_y , respectively and output at ports **3** and **4** have annihilation operators $\hat{c}_{x,y}$ and $\hat{d}_{x,y}$, respectively, for the two mode coherent state $|\alpha_x, \alpha_y\rangle$ we have

$$\hat{a}_x|\alpha_x, \alpha_y\rangle = \alpha_x|\alpha_x, \alpha_y\rangle, \hat{a}_y|\alpha_x, \alpha_y\rangle = \alpha_y|\alpha_x, \alpha_y\rangle, \quad (7)$$

and for two mode squeezed vacuum

$$\hat{b}_x(t) = c \hat{b}_x|0\rangle + is \hat{b}_y^\dagger|0\rangle, \hat{b}_y(t) = c \hat{b}_y|0\rangle + is \hat{b}_x^\dagger|0\rangle, \quad (8)$$

where $c = \cosh kt$, $s = \sinh kt$, kt being the interaction time for nonlinear interaction. After this superposition, the beams at ports **3** and **4** can be represented as

$$\hat{c} = t\hat{a} + ir\hat{b}, \hat{d} = t\hat{a} + ir\hat{a}, \quad (9)$$

where t and r are the transmission coefficient and reflection coefficients, respectively.

In this problem, we are only interested in port **3** and we can write the x and y modes of the annihilation operator at the output port **3** as

$$\hat{c}_x = t_x \hat{a}_x + ir_x \hat{b}_x, \hat{c}_y = t_y \hat{a}_y + ir_y \hat{b}_y, \quad (10)$$

with $t_{x,y}$ and $r_{x,y}$ being the transmission coefficient and reflection coefficient, respectively for the two modes.

To have an idea about the non-classicality in mixing of two light beams, let us consider, the two input beams having density operators

$$\hat{\rho}_1 = \int d^2\alpha P_1(\alpha)|\alpha\rangle\langle\alpha|, \hat{\rho}_2 = \int d^2\beta P_2(\beta)|\beta\rangle\langle\beta|. \quad (11)$$

The composite density operator can therefore be written as

$$\begin{aligned} \hat{\rho} &= \int d^2\alpha d^2\beta P_1(\alpha)P_2(\beta)|\alpha, \beta\rangle\langle\alpha, \beta| \\ &= \int d^2\alpha d^2\beta P_1(\alpha)P_2(\beta) \exp[-(|\alpha|^2 + |\beta|^2)] \\ &\quad \exp[\alpha a^\dagger + \beta b^\dagger - h.c.]|0,0\rangle\langle 0,0| \exp[\alpha^* a + \beta^* b - h.c.], \end{aligned} \quad (12)$$

where $h.c.$ stands for hermitian conjugate. This leads to

$$\alpha \hat{a}^\dagger + \beta \hat{b}^\dagger = (t\alpha - ir\beta)\hat{c}^\dagger + (t\beta - ir\alpha)\hat{d}^\dagger, \quad (13)$$

and hence

$$\begin{aligned} \hat{\rho} &= \int d^2\alpha d^2\beta P_1(\alpha)P_2(\beta)|t\alpha - ir\beta, t\beta - ir\alpha\rangle \\ &\quad \langle t\alpha - ir\beta, t\beta - ir\alpha| \\ &= \int d^2r d^2\delta P(r, \delta)|r, \delta\rangle\langle r, \delta|, \end{aligned} \quad (14)$$

where $P(R, \delta) = P_1(tr + ir\delta) P_2(t\delta + ir\delta)$.

This shows that, if P_1 and P_2 are non negative, P also has a non-negative value and classical input light beams mix at a linear beam splitter to generate classical output light beams. However, if one of P_1 and P_2 is not non-negative, P would also be non-negative and input non-classical beam gives non-classicality in the output. Generation of anti-bunched light by mixing of squeezed light with classical light is a very well known example. We observe here the non-classicality in the form of polarization squeezing exhibited here in a similar manner.

THE TWO MODE SQUEEZED VACUUM AND POLARIZATION SQUEEZING

The two mode squeezed vacuum as shown in the Eq. (8) can also be represented as,

$$\hat{b}_x(t) = c\hat{b}_{x0}|0\rangle + is\hat{b}_{y0}^\dagger|0\rangle, \hat{b}_y(t) = c\hat{b}_{y0}|0\rangle + is\hat{b}_{x0}^\dagger|0\rangle, \quad (15)$$

where b_{x0}, b_{y0} are the annihilation operators initially and $b_x(t), b_y(t)$ at time t after the non-degenerate parametric amplification. For initial vacuum state, straight calculations give

$$\begin{aligned} \langle \hat{S}_0 \rangle &= 2s^2, \langle \hat{S}_1 \rangle = \langle \hat{S}_2 \rangle = \langle \hat{S}_3 \rangle = 0, \\ \langle \hat{S}_1^2 \rangle &= \langle \hat{S}_2^2 \rangle = \langle \hat{S}_3^2 \rangle = 4s^4 + 4s^2. \end{aligned} \quad (16)$$

It is easy to see by plugging in the values in the inequality criterion for polarization squeezing given by Eq. (5) that, none

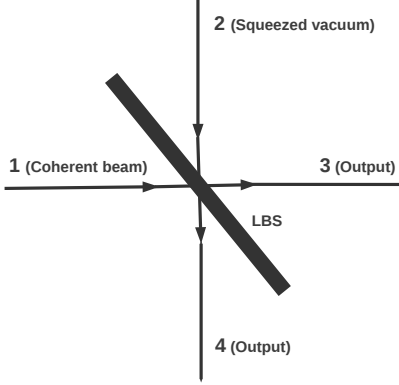


FIG. 1. Superposition of coherent light beam and squeezed vacuum through linear polarization beam splitter.

of the Stokes operators show polarization squeezing. But, it is very important to notice that the squeezed vacuum is observed squeezed in polarization[14] when the first criterion for polarization squeezing [5] is used, however, the two mode squeezed vacuum shows no squeezing in polarization on using the general criterion that considers uncertainty relation followed by Stokes operators in addition to the commutation relations. We observe that it does not exhibit any squeezing in polarization as per the general criteria [7, 8] for a general component of Stokes operator vector.

POLARIZATION SQUEEZING IN THE MIXED BEAM THROUGH LINEAR BEAM SPLITTER

After mixing of the coherent beam and squeezed vacuum, the mean values and variances of the Stokes operators at the output port **3** represented by Eq. (10) for $\alpha_{x,y} = |\alpha_{x,y}|e^{i\phi_{x,y}}$ can be calculated as

$$\begin{aligned}\langle S_0 \rangle &= t_x^2 |\alpha_x|^2 + t_y^2 |\alpha_y|^2 + (r_x^2 - r_y^2) s^2, \\ \langle S_1 \rangle &= t_x^2 |\alpha_x|^2 - t_y^2 |\alpha_y|^2 + (r_x^2 + r_y^2) s^2, \\ \langle S_2 \rangle &= 2t_x t_y |\alpha_x| |\alpha_y| \cos(\phi_y - \phi_x), \\ \langle S_3 \rangle &= 2t_x t_y |\alpha_x| |\alpha_y| \sin(\phi_y - \phi_x),\end{aligned}\quad (17)$$

and

$$\begin{aligned}V_1 &= 2s^2 r_x^2 t_x^2 |\alpha_x|^2 + 2s^2 r_y^2 t_y^2 |\alpha_y|^2 + t_x^2 |\alpha_x|^2 + t_y^2 |\alpha_y|^2 \\ &\quad - 4csr_x r_y t_x t_y |\alpha_x| |\alpha_y| \sin(\phi_x + \phi_y) \\ &\quad + (r_x^2 + r_y^2) s^2 - (r_x^2 - r_y^2) s^4, \\ V_2 &= 2s^2 t_x^2 r_y^2 |\alpha_x|^2 + 2s^2 r_x^2 t_y^2 |\alpha_y|^2 + t_x^2 |\alpha_x|^2 + t_y^2 |\alpha_y|^2 \\ &\quad - 4csr_x r_y t_x t_y |\alpha_x| |\alpha_y| \sin(\phi_x + \phi_y) \\ &\quad + 4r_x^2 r_y^2 s^4 + 2r_x^2 r_y^2 (2s^4 + s^2), \\ V_3 &= 2s^2 t_x^2 r_y^2 |\alpha_x|^2 + 2s^2 r_x^2 t_y^2 |\alpha_y|^2 + t_x^2 |\alpha_x|^2 + t_y^2 |\alpha_y|^2 \\ &\quad - 4csr_x r_y t_x t_y |\alpha_x| |\alpha_y| \sin(\phi_x + \phi_y) \\ &\quad - 4r_x^2 r_y^2 s^4 + 2r_x^2 r_y^2 (2s^4 + s^2),\end{aligned}\quad (18)$$

We now consider the polarization squeezing under the approximation $t_x |\alpha_x| = t_y |\alpha_y| = A \gg c$, i.e., the transmitted parts of coherent light in the x and y modes have the same amplitude which is very large as compared to $\cosh kt$. This is to note that, if we do not consider large interaction times which allows us to ignore the higher order terms in s , i.e. s^2 and s^4 , we can test the polarization squeezing along all the three Stokes operators as shown below. For the Stokes operator \hat{S}_1 , we have

$$\begin{aligned}V_1 &= 2s^2 (r_x^2 + r_y^2) A^2 - 4csr_x r_y A^2 \sin(\phi_x + \phi_y) + 2A^2, \\ \langle \hat{S}_2 \rangle^2 + \langle \hat{S}_3 \rangle^2 &= 8A^4.\end{aligned}$$

The squeezing factor for \hat{S}_1 obtained by plugging in these values in Eq. (6) is

$$S_1 = \frac{1}{\sqrt{2}} [1 + s^2 (r_x^2 + r_y^2) - 2csr_x r_y \sin(\phi_x + \phi_y)]. \quad (19)$$

This expression has the minimum value for $\phi_x + \phi_y = \pi/2$ and the maximum polarization squeezing would be obtained for $\tanh 2kt = 2r_x r_y / (r_x^2 + r_y^2)$. Therefore, the maximum polarization squeezing quantified by the minimum value of polarization squeezing factor is

$$S_{1min} = \frac{1}{\sqrt{2}} \left[1 - [\min(r_x, r_y)]^2 \right], \quad (20)$$

where $\min(r_x, r_y)$ is r_x if $r_x < r_y$, r_y if $r_y < r_x$ and $r_y = r_x$ if both are equal.

For second Stokes operator \hat{S}_2 as per the same criterion in Eq. (6), squeezing factor can be written as

$$S_2 = \frac{1 + s^2 (r_x^2 + r_y^2) - 2csr_x r_y \sin(\phi_x + \phi_y)}{\sin(\phi_y - \phi_x)}. \quad (21)$$

This expression can be minimized for $(\phi_x + \phi_y) = \pi/2$ and $(\phi_y - \phi_x) = \pi/2$, and it leads to maximum squeezing for $\tanh kt = 2r_x r_y / (r_x^2 + r_y^2)$ with squeezing factor resulting into the same expression as in the previous case. It is given by

$$S_{2min} = 1 - [\min(r_x, r_y)]^2. \quad (22)$$

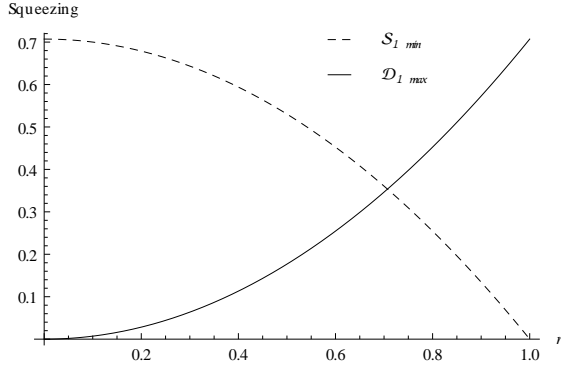


FIG. 2. Variation of S_{1min} and D_{1max} with r showing maximum polarization squeezing.

In a similar way, we obtain the minimum squeezing factor in case of Stokes operator \hat{S}_3 for $(\phi_x + \phi_y) = \pi/2$ and $(\phi_y - \phi_x) = 0$ with a condition $\tanh kt = 2r_x r_y / r_x^2 + r_y^2$, as

$$S_{3min} = 1 - [\min(r_x, r_y)]^2. \quad (23)$$

The squeezing factor for all the three components of Stokes operator vector can therefore be written as

$$S_{1min} = \frac{1}{\sqrt{2}}[1 - r^2], S_{2min} = S_{3min} = 1 - r^2, \quad (24)$$

where $r = \min(r_x, r_y)$.

Above results show that the squeezed vacuum is not polarization squeezed in itself, but when mixed with a coherent beam, the output beam exhibits polarization squeezing along all the three components of Stokes operator vector for $(\phi_x + \phi_y) = \pi/2$ but different combinations of (ϕ_x, ϕ_y) . We observe that the output light is most squeezed in polarization along the Stokes parameters \hat{S}_1 and equally squeezed along \hat{S}_2 and \hat{S}_3 . The variation of squeezing factor and degree of squeezing with r , corresponding to maximum polarization squeezing in \hat{S}_2 and \hat{S}_3 is shown in Fig. 2.

DISCUSSION OF RESULT

Looking at the above expressions for squeezing factor, one can observe that, it can be made less than one by choosing r small. This gives high degree of polarization squeezing. As a special case, if we consider this linear beam splitter to be symmetric one, $r_x = r_y = \frac{1}{\sqrt{2}}$ that gives the minimum squeezing factor $S_{1min} = 0.35$, i.e., $D_{1min} = 0.65$. However, in the

case of \hat{S}_2 and \hat{S}_3 , we have $S_{2min} = S_{3min} = 0.35$, and $D_{2max} = D_{3min} = 0.50$. This reveals a maximum of 65% squeezing at the output port 3 and this is observed along \hat{S}_1 . We therefore observe that squeezed vacuum on mixing with a coherent radiation through a beam splitter leads to polarization squeezing in the output light. This is important because the nature of non-classicality changes during the interaction and up to 65% squeezing in polarization is obtained without initial non-classicality of same nature. In our next manuscript, we are further exploring the simultaneous squeezing of orthogonal components of Stokes operator vector.

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